Reissner-Nordström-AdS black hole in the GEMS approach

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We obtain a (5+2)-dimensional global flat embedding of the (3+1)-dimensional curved RN-AdS space. Our results include the various limiting cases of global embedding Minkowski space (GEMS) geometries of the RN, Schwarzschild-AdS space-time in 5+2 dimensions, Schwarzschild space-time in 5+1 dimensions, purely charged space, and universal covering space of AdS space-time in 4+1 dimensions, through the successive truncation procedure of parameters in the original curved space.

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I. INTRODUCTION

Ever since the discovery that thermodynamic properties of black holes in anti–de Sitter (AdS) space-time are dual to those of a field theory in one dimension fewer, there has been much interest in the Reissner-Nordström- (RN-) AdS black hole [1], which now becomes a prototype example to study this AdS/conformal field theory (CFT) correspondence [2]. On the other hand, after Unruh's work [3], it has been known that the thermal Hawking effect on a curved manifold [4] can be looked at as an Unruh effect in a higher dimensional flat space-time. Recently, nontrivial works of isometric embeddings of the Reissner-Nordström (RN) black hole [5] and M2-, D3-, M5-branes [6] into flat spaces with two times have been studied to get some insight into the global aspect of the space-time geometries in the context of brane physics. Moreover, several authors [7–9] have also shown that the global embedding Minkowski space (GEMS) approach [10–14] of which a hyperboloid in a higher dimensional space corresponds to original curved space could provide a unified derivation of temperature for a wide variety of curved spaces. These include the static, rotating, charged Bañaos-Teitelboim-Zanelli (BTZ) [15–17], the Schwarzschild [18] together with its AdS extensions, and the RN [19] black holes. Therefore, it is interesting to study the geometry of the RN-AdS and their thermodynamics [20] in this GEMS approach.

In this paper we will analyze the Hawking and Unruh effects of the $D=4$ RN-AdS space, which has not been tackled up to now due to the complicated structure of this system, in terms of the GEMS approach covering the usual Kruskal extension [21]. In Sec. II, we discuss the $D=4$ RN-AdS embedding into a seven dimensional flat space. In Sec. III, we show that our results in the GEMS of the RN-AdS space systematically include those of the various limiting GEMS geometries, which are the RN, Schwarzschild-AdS, Schwarzschild, purely charged and AdS space-times, through the successive truncation procedure of parameters in the original curved space. These correspond to the dimensional reduction in the GEMS approach. Finally, we present a summary in Sec. IV.

II. GEOMETRIC STRUCTURE OF RN-ADS IN THE GEMS APPROACH

Let us consider the line element of the four dimensional RN-AdS space[1] [1]

$$ds_4^2 = f(r,m,e,R)dt^2 - f^{-1}(r,m,e,R)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where $f(r,m,e,R)$ is given by

$$f(r,m,e,R) = 1 - \frac{2m}{r} + \frac{e^2}{r^2} + \frac{r^2}{R^2}. \tag{2}$$

This space-time is asymptotically described by AdS, and there is an outer horizon at $r=r_H$. The case of $e=0$ yields the Schwarzschild-AdS metric, the case of $m=e=0$ yields the metric on the universal covering space of AdS [22], the case of $R\to\infty$ yields the RN metric, and the case of $m=0$ and $R\to\infty$ yields the purely charged metric.

To embed this space-time into a higher dimensional flat one, we first note that by introducing three coordinates (z^3,z^4,z^5) in Eq. (10) (see below) the last term in the metric

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[1]We restrict our discussion to the nonextremal case.

(1) can be written to give $-(dz^3)^2-(dz^4)^2-(dz^5)^2=-dr^2-r^2(d\theta^2+\sin^2\theta d\phi^2)$. Then, making use of an ansatz of two coordinates (z^0, z^1) in Eq. (10), we have obtained

$$\begin{aligned}(dz^0)^2-(dz^1)^2-(dz^3)^2-(dz^4)^2-(dz^5)^2&=f(r,m,e,R)dt^2-r^2(d\theta^2+\sin^2\theta d\phi^2)\\&-\left(1+\frac{\left(\frac{f'(r,m,e,R)}{2}\right)^2}{k_H^2 f(r,m,e,R)}\right)dr^2,\end{aligned}\tag{3}$$

where $f'(r,m,e,R)$ denotes the derivative with respect to r and

$$k_H(r_H,e,R^2)=\frac{\left(r_H^2-e^2+\frac{3r_H^4}{R^2}\right)}{2r_H^3}>0\tag{4}$$

is the surface gravity at the root of $f(r,m,e,R)|_{r=r_H}=0$. In order to make the form of ds_4^2 in Eq. (1), we subtract the $f^{-1}(r,m,e,R)dr^2$ term from Eq. (3) on the right-hand side and add it again to Eq. (3). Then, the remaining extra radial part of

$$f^{-1}(r,m,e,R)dr^2-\left(1+\frac{\left(\frac{f'(r,m,e,R)}{2}\right)^2}{k_H^2 f(r,m,e,R)}\right)dr^2\tag{5}$$

can be separated into positive and negative definite parts with $r>r_H$ as follows:

$$\begin{aligned}&R\left(\frac{e^2}{[rr_H(r^2+rr_H+r_H^2)+(rr_H-e^2)R^2]}+\frac{r_H^2(r^2+rr_H+r_H^2)[(r_H^2-e^2)^2R^4+r_H^6(r_H^2+2R^2)]}{r^2[3r_H^4+(r_H^2-e^2)R^2]^2[rr_H(r^2+rr_H+r_H^2)+(rr_H-e^2)R^2]}\right)dr^2\\&-\left(e^2\frac{R^4r_H^6[4(rr_H-e^2)R^2+10r^4+2rr_H(r^2+rr_H+2r_H^2)]}{r^4[3r_H^4+(r_H^2-e^2)R^2]^2[rr_H(r^2+rr_H+r_H^2)+(rr_H-e^2)R^2]}\right)dr^2\\&-\left(\frac{rr_H(r^2+rr_H+r_H^2)(4r_H^6R^2+[3r_H^4+(r_H^2-e^2)R^2]^2)}{[3r_H^4+(r_H^2-e^2)R^2]^2[rr_H(r^2+rr_H+r_H^2)+(rr_H-e^2)R^2]}\right)dr^2=(dz^2)^2-(dz_e)^2-(dz_R)^2,\end{aligned}\tag{6}$$

where we have used the relation between the Arnowitt-Deser-Misner mass of the RN-AdS black hole and its event horizon radius $r=r_H$, i.e., $2m=r_H+r_H^3/R^2+e^2/r_H$. At this stage, it should be note that due to the existence of the last two terms, e-sensitive $(dz_e)^2$ and R-dominant $(dz_R)^2$, one may think that superficially two additional time dimensions are needed for a global flat embedding. However, it is in fact enough to introduce only one time dimension $(dz^6)^2$ by combining these two terms as

$$(dz^6)^2=(dz_e)^2+(dz_R)^2,\tag{7}$$

for a desired minimal[2] GEMS with an additional spacelike dimension $(dz^2)^2$. Note also that the $(dz_e)^2$ [or, $(dz_R)^2$] term is shown to be vanished in the limit of $e\to 0$ (or, $R\to\infty$), and the $(dz^6)^2$ becomes $(dz_R)^2$ [or, $(dz_e)^2$]. As a result, we have obtained a flat global embedding in (5+2)-dimensions of the corresponding curved 4-metric as

$$\begin{aligned}ds_7^2&=(dz^0)^2-\sum_{i=1}^{5}(dz^i)^2+(dz^6)^2\\&=f(r,m,e,R)dt^2-f^{-1}(r,m,e,R)dr^2\\&\quad-r^2(d\theta^2+\sin^2\theta d\phi^2)\end{aligned}\tag{8}$$

$$=ds_4^2.\tag{9}$$

This equivalence between the (5+2)-dimensional flat embedding space and original curved space is the very definition of isometric embedding, mathematically developed by several authors [6,23].

It seems appropriate to comment on the lowest embedding dimensions in terms of the number of parameters. It is known from the previous works [5,13,17] that whenever one parameter is increased in the original space, the embedding dimensions are either unchanged or increased depending on this parameter. In particular, the embedding dimension is already $D=7$ for the case of the RN or Schwarzschild-AdS, which have one less parameters than those of the RN-AdS case. Therefore, for the case of the RN-AdS the possibly lowest embedding dimension is $D=7$.

In summary, through the GEMS approach which makes the curved space possibly embedded in a higher dimensional

[2]In the region of $r>r_H$, it can be easily verified that the $(dz^2)^2$ and $(dz^6)^2$ are positive definite functions, when combined with the condition in Eq. (4).

flat space [10–14], we have found a $D=7$ dimensional isometric embedding of the RN-AdS space as

$$z^0=k_H^{-1}\sqrt{f(r,m,e,R)}\,\sinh(k_H t),$$

$$z^1=k_H^{-1}\sqrt{f(r,m,e,R)}\,\cosh(k_H t),$$

$$z^2=\int dr R\left(\frac{e^2}{[rr_H(r^2+rr_H+r_H^2)+(rr_H-e^2)R^2]}+\frac{r_H^2(r^2+rr_H+r_H^2)[(r_H^2-e^2)^2R^4+r_H^6(r_H^2+2R^2)]}{r^2[3r_H^4+(r_H^2-e^2)R^2]^2[rr_H(r^2+rr_H+r_H^2)+(rr_H-e^2)R^2]}\right)^{1/2},$$

$$z^3=r\sin\theta\cos\phi,$$

$$z^4=r\sin\theta\sin\phi,$$

$$z^5=r\cos\theta,$$

$$z^6=\int dr\left(\frac{e^2R^4r_H^6[4(rr_H-e^2)R^2+10r^4+2rr_H(r^2+rr_H+2r_H^2)]}{r^4[3r_H^4+(r_H^2-e^2)R^2]^2[rr_H(r^2+rr_H+r_H^2)+(rr_H-e^2)R^2]}\right.$$
$$\left.+\frac{rr_H(r^2+rr_H+r_H^2)(4r_H^6R^2+[3r_H^4+(r_H^2-e^2)R^2]^2)}{[3r_H^4+(r_H^2-e^2)R^2]^2[rr_H(r^2+rr_H+r_H^2)+(rr_H-e^2)R^2]}\right)^{1/2}, \tag{10}$$

with additional spacelike z^2 and timelike z^6 dimensions. Therefore, the $(3+1)$-dimensional curved space is seen as the hyperboloid embedded in a $(5+2)$-dimensional flat space. It would be easily verified inversely that the flat metric (8) in the $(5+2)$-dimensional space defined as the coordinates (10) gives the original RN-AdS metric (1) correctly.

Now, following the trajectory of $z^2=\cdots=z^6=0$ in Eq. (10) which corresponds to a static trajectory $(r,\theta,\phi=\text{const})$ in the curved space, the relevant $D=7$ acceleration a_7 is described as the Rindler-like motion [7,14,17] of the form of $(z^1)^2-(z^0)^2=a_7^{-2}$ in the embedded flat space, i.e.,

$$a_7=\{(z^1)^2-(z^0)^2\}^{-1/2}=\frac{r_H^2-e^2+\dfrac{3r_H^4}{R^2}}{2r_H^3\sqrt{f(r,m,e,R)}}. \tag{11}$$

As a result, the detector of the above Rindler-like motion would measure the correct Hawking temperature through the relation of $T=a_7/2\pi$ as follows:

$$T=\frac{r_H^2-e^2+\dfrac{3r_H^4}{R^2}}{4\pi r_H^3\sqrt{f(r,m,e,R)}}, \tag{12}$$

in the GEMS approach. Then, the desired BH temperature is given by

$$T_0=\sqrt{g_{00}}T=\frac{r_H^2-e^2+\dfrac{3r_H^4}{R^2}}{4\pi r_H^3}. \tag{13}$$

It is by now well known that entropy, which is the extensive companion of the temperature, is given by one quarter of the horizon area [24]. On the other hand, Laflamme [25] showed that entropy seen by an accelerated observer in Minkowski space can be obtained from the consideration of the transverse area of a null surface on the wedge. This transverse area would diverge for otherwise unrestricted Rindler motion due to the integration over the whole transverse dimensions. In an embedded higher dimensional flat space, however, since there are ''embedding'' constraints, the resulting integral may not be divergent and make entropy finite.

Our RN-AdS case, where there are three additional dimensions in the transverse area, $\int dz^2\ldots dz^6$, is correspondingly subject to four constraints as follows:

$$(z^1)^2-(z^0)^2=0, \tag{14}$$

$$z^2=f_1(r),\quad z^6=f_2(r), \tag{15}$$

$$(z^3)^2+(z^4)^2+(z^5)^2=r^2, \tag{16}$$

where $f_i(r)$ are explicitly given in Eq. (10). Note that Eq. (14) leads to $r=r_H$. Since the z^2 and z^6 integrals subject to the constraints (15), $\int dz^2dz^6\delta(z^2-f_1(r))\delta(z^6-f_2(r))$, is unity, the remaining integrals of $z_i(i=3,4,5)$ well reproduce the desired area $4\pi r_H^2$ of the $r=r_H$ sphere. This ends the global flat embedding of the RN-AdS space giving the correct thermodynamics.

III. VARIOUS LIMITING GEOMETRIES

Now, we are ready to analyze the various limiting geometries through the successive truncation procedure of the parameters, e, or R (or, both) in the original curved space.

A. RN limit

Let us first consider the RN limit [19,26], which is the case of $R\to\infty$ in the metric (1),

$$ds_4^2=f_e(r,m,e)dt^2-f_e^{-1}(r,m,e)dr^2-r^2(d\theta^2+\sin^2\theta d\phi^2), \tag{17}$$

where

$$f_e(r,m,e)=f(r,m,e,R\to\infty)\equiv 1-\frac{2m}{r}+\frac{e^2}{r^2}. \tag{18}$$

The global flat embedding coordinates can be obtained either in the GEMS approach [7] starting from the RN metric, Eq. (17), or in the limit of $R\to\infty$ from Eq. (10) directly as

$$\begin{aligned}
z^0&=k_H^{-1}\sqrt{f_e(r,m,e)}\sinh(k_Ht),\\
z^1&=k_H^{-1}\sqrt{f_e(r,m,e)}\cosh(k_Ht),\\
z^2&=\int dr\left(\frac{r^2(r_++r_-)+r_+^2(r+r_+)}{r^2(r-r_-)}\right)^{1/2},\\
z^3&=r\sin\theta\cos\phi,\\
z^4&=r\sin\theta\sin\phi,\\
z^5&=r\cos\theta,\\
z^6&=\int dr\left(\frac{4r_+^5r_-}{r^4(r_+-r_-)^2}\right)^{1/2},
\end{aligned} \tag{19}$$

where the surface gravity is given by $k_H=k_H(r_H,e,\infty)=(r_+-r_-)/2r_+^2$ with the outer horizon $r_+=r_H$, and $r_\pm=m\pm\sqrt{m^2-e^2}$. In this limit, the R-dominant part of z^6 in Eq. (10) vanishes and the resulting GEMS becomes exactly the known $D=7$ RN one [7]. Note that in the limit of $R\to\infty$ the corresponding event horizon becomes the usual RN one by rewriting the charge e^2 to r_+r_-.

Moreover, the relevant $D=7$ acceleration and the Hawking temperature can be obtained either directly from Eqs. (11) and (12) by taking the limit of $R\to\infty$ and replacing e^2 with r_+r_-, or from the Rindler-like motion in the $D=7$ GEMS, Eq. (19), following a static trajectory $(r,\theta,\phi=\text{const})$ in the curved space as before:

$$a_7=\{(z^1)^2-(z^0)^2\}^{-1/2}=\frac{r_+-r_-}{2r_+^2\sqrt{f_e(r,m,e)}}, \tag{20}$$

$$T=\frac{r_+^2-e^2}{4\pi r_+^3\sqrt{f_e(r,m,e)}}=\frac{r_+-r_-}{4\pi r_+^2\sqrt{f_e(r,m,e)}}. \tag{21}$$

The entropy calculation of the RN is essentially the same as the previous RN-AdS case. In this case there are three additional dimensions, and four constraints, i.e., $(z^1)^2-(z^0)^2=0$ leads to $r=r_+$, $z^2=f_1(r,R\to\infty)$, $z^6=f_2(r,R\to\infty)$ in Eqs. (10) and $(z^3)^2+(z^4)^2+(z^5)^2=r^2$. Thus, since the z^2,z^6 integrals, $\int dz^2dz^6\delta(z^2-f_1(r))\delta(z^6-f_2(r))$, are unity, the remaining integrals give the desired area $4\pi r_H^2$, that of the corresponding $r=r_H$ sphere.

B. Schwarzschild-AdS limit

Secondly, the RN-AdS solution (10) is also easily reduced to the Schwarzschild-AdS space, which is the limiting case of $e\to 0$,

$$\begin{aligned}
ds_4^2=&f_R(r,m,R)dt^2-f_R^{-1}(r,m,R)dr^2\\
&-r^2(d\theta^2+\sin^2\theta d\phi^2),
\end{aligned} \tag{22}$$

where

$$f_R(r,m,R)=f(r,m,e=0,R)\equiv 1-\frac{2m}{r}+\frac{r^2}{R^2}, \tag{23}$$

giving another $D=7$ GEMS with the vanishing e-sensitive part of z^6 in Eq. (10),

$$\begin{aligned}
z^0&=k_H^{-1}\sqrt{f_R(r,m,R)}\sinh(k_Ht),\\
z^1&=k_H^{-1}\sqrt{f_R(r,m,R)}\cosh(k_Ht),\\
z^2&=\int dr\frac{R^3+Rr_H^2}{R^2+3r_H^2}\sqrt{\frac{r_H(r^2+rr_H+r_H^2)}{r^3(r^2+rr_H+r_H^2+R^2)}},\\
z^3&=r\sin\theta\cos\phi,\\
z^4&=r\sin\theta\sin\phi,\\
z^5&=r\cos\theta,\\
z^6&=\int dr\sqrt{\frac{(R^4+10R^2r_H^2+9r_H^4)(r^2+rr_H+r_H^2)}{(R^2+3r_H^2)^2(r^2+rr_H+r_H^2+R^2)}}.
\end{aligned} \tag{24}$$

The surface gravity, $k_H=k_H(r_H,0,R)=(R^2+3r_H^2)/2r_HR^2$, is now either obtained at the root r_H of $f_R(r,m,R)|_{r=r_H}=0$, or reduced directly from the Eq. (4) with $e=0$. This seemingly complicated embedding space is first obtained in Ref. [7], and we have also reached to the exactly same results by the systematic reduction process from Eq. (10).

On the other hand, similar to the RN limit case, we directly obtain the Hawking temperature from Eqs. (11) and (12) by taking the limit of $e\to 0$ as follows:

$$T=\frac{a_7}{2\pi}=\frac{1+\dfrac{3r_H^2}{R^2}}{4\pi r_H\sqrt{f_R(r,m,R)}}, \tag{25}$$

which again equals to that calculated in [27].

C. Schwarzschild limit

Thirdly, we can obtain the Schwarzschild limit without the cosmological constant from the RN embedding of Eq. (19) with $e\to 0$ limit or the Schwarzschild-AdS embedding of Eq. (24) with $R\to\infty$ one. As a result, it is successfully reduced to the $D=6$ flat GEMS as follows [11]:

$$z^0=k_H^{-1}\sqrt{1-2m/r}\,\sinh(k_H^{-1}t),$$

$$z^1=k_H^{-1}\sqrt{1-2m/r}\,\cosh(k_H^{-1}t),$$

$$z^2=\int dr\,\sqrt{r_H(r^2+rr_H+r_H^2)/r^3},$$

$$z^3=r\sin\theta\sin\phi,$$

$$z^4=r\sin\theta\cos\phi,$$

$$z^5=r\cos\theta, \tag{26}$$

where the event horizon is $r_H=2m$, and the surface gravity is $k_H(r_H,0,\infty)=1/2r_H$. Note that the analyticity of $z^2(r)$ in $r>0$ covers the region of $r<r_H$. Thus, it should be cautioned that the use of incomplete embedding spaces, that cover only $r>r_H$ (as, for example, in [13]), will lead to observers there for whom there is no event horizon, no loss of information, and no temperature.

We then obtain the Hawking temperature from Eqs. (20) and (21) by taking the limit $e\to 0$ as follows:

$$T=\frac{a_6}{2\pi}=\frac{1}{8\pi m\sqrt{1-2m/r}},$$

$$T_0=\sqrt{g_{00}}T=\frac{1}{8\pi m}. \tag{27}$$

It seems appropriate to comment on a global flat embedding of $D=4$ covering of the AdS,

$$ds_4^2=\left(1+\frac{r^2}{R^2}\right)dt^2-\left(1+\frac{r^2}{R^2}\right)^{-1}dr^2 -r^2(d\theta^2+\sin^2\theta d\phi^2), \tag{28}$$

which corresponds to the case of $m\to 0$ in Eq. (22). In this case we cannot directly obtain a global embedding from Eq. (24) since in the limit of $m\to 0$ the surface gravity $k_H=1/2r_H=1/4m$ yields a divergence. As discussed in Ref. [7] in details, this problem originally comes from the fact that there is no intrinsic horizon of this space-time. However, there is of course the other direct route to embed this space-time into the $D=5$ flat space-time starting from the metric (22) with $m=0$. Based on the accelerating coordinate system, the correct temperature of $2\pi T=(a^2-R^{-2})^{1/2}$ has been already found. (For further details, see Ref. [7].)

Similar to the pure AdS case, we can directly analyze the purely charged case with $m=0$ in the metric (17) as

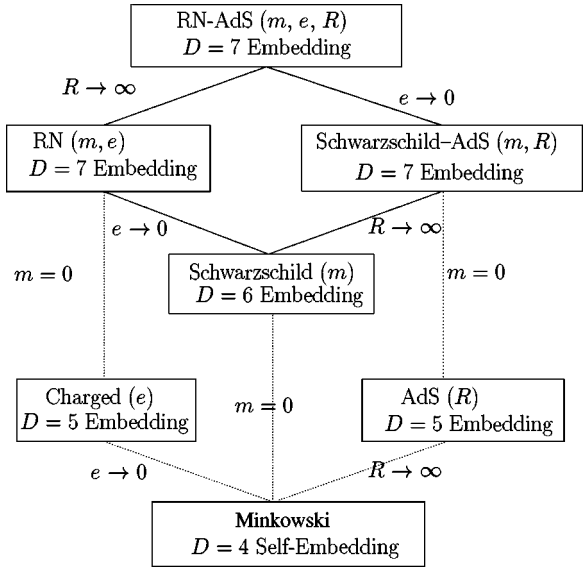

FIG. 1. Various limits of RN-AdS embedding: Trunction connected by solid lines means that through the direct parameter reductions it is possible to obtain all thermodynamic quantities in the lower dimensional embedding system, while truncation connected by dotted lines means that these parameter reductions are only possible at the level of metric.

$$ds_4^2=\left(1+\frac{e^2}{r^2}\right)dt^2-\left(1+\frac{e^2}{r^2}\right)^{-1}dr^2-r^2(d\theta^2+\sin^2\theta d\phi^2). \tag{29}$$

As like the above $D=4$ covering of the AdS, this has also no event horizon. However, we can also embed this space-time into $D=5$ flat one in view of the accelerating coordinate frame as follows:

$$z^0=\sqrt{\rho^2-e^2}\,\sinh(\eta/e),$$

$$z^1=\sqrt{\rho^2-e^2}\,\cosh(\eta/e),$$

$$z^2=\rho\sinh\Phi\cos\theta,$$

$$z^3=\rho\sinh\Phi\sin\theta,$$

$$z^4=\rho\cosh\Phi, \tag{30}$$

where $-\infty<\eta,\Phi<\infty,-\pi<\theta<\pi$. While this coordinate patch only covers the region $\rho>e$, it can be extended to the entire space similar to the four dimensional AdS case [7]. Then, we can easily obtain the temperature as $2\pi T=(a^2-e^{-2})^{1/2}$ where the four acceleration a is given by $a=\rho^2/e^2(\rho^2-e^2)$.

Furthermore, if we take the limit $R\to\infty$ in the metric (28), or $e\to 0$ in the metric (29), we finally reach to the flat four dimensional Minkowski space.

We have summarized all these results in Fig. 1 as a compact diagram, which can be obtained through the systematic truncation procedure from the seven dimensional flat embedding space.

IV. SUMMARY

In summary, we have shown that the Hawking thermal properties map into their Unruh equivalents in the $(5+2)$-dimensional GEMS, which is the lowest possible global embedding dimensions of the curved RN-AdS space. The relevant curved space detectors become Rindler ones, whose temperatures and entropies reproduce the originals. Our results of the RN-AdS in the GEMS approach include the various limiting geometries, which are the Reissner-Nordström, Schwarzschild-AdS, and Schwarzschild space-times through the successive reduction procedure of the parameters in the original space. As a result, the $(5+2)$-dimensional GEMS in Eq. (10) serves a unifying description of the global flat embedding of the various geometries. It would be interesting to consider other interesting applications of the GEMS, for example, the rotating Kerr type geometries [15,16,28,29].

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[1] B. Carter, in *Black Holes*, edited by C. M. DeWitt and B. S. DeWitt (Gordon and Breach, New York, 1973).
[2] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).
[3] W. G. Unruh, Phys. Rev. D **14**, 870 (1976); P. C. W. Davies, J. Phys. A **8**, 609 (1975).
[4] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1974).
[5] S. Deser and O. Levin, Phys. Rev. D **59**, 064004 (1999).
[6] L. Andrianopoli, M. Derix, G. W. Gibbons, C. Herdeiro, A. Santambrogio, and A. Van Proeyen, Class. Quantum Grav. **17**, 1875 (2000).
[7] S. Deser and O. Levin, Class. Quantum Grav. **14**, L163 (1997); **15**, L85 (1998).
[8] M. Beciu and H. Culetu, Mod. Phys. Lett. A **14**, 1 (1999).
[9] P. F. Gonzalez-Diaz, Phys. Rev. D **61**, 024019 (2000).
[10] E. Kasner, Am. J. Math. **43**, 130 (1921).
[11] C. Fronsdal, Phys. Rev. **116**, 778 (1959).
[12] A. Friedman, Rev. Mod. Phys. **37**, 201 (1965); H. F. Goenner, in *General Relativity and Gravitation*, edited by A. Held (Plenum, New York, 1980).
[13] J. Rosen, Rev. Mod. Phys. **37**, 204 (1965).
[14] H. Narnhofer, I. Peter, and W. Thirring, Int. J. Mod. Phys. B **10**, 1507 (1996).
[15] M. Banados, C. Teitelboim, and J. Zanelli, Phys. Rev. Lett. **69**, 1849 (1992).
[16] S. W. Kim, W. T. Kim, Y.-J. Park, and H. Shin, Phys. Lett. B **392**, 311 (1997).
[17] S.-T. Hong, Y.-W. Kim, and Y.-J. Park, Phys. Rev. D **62**, 024024 (2000); S.-T. Hong, W. T. Kim, Y.-W. Kim, and Y.-J. Park, *ibid.* **62**, 064021 (2000).
[18] K. Schwarzschild, Sitzber. Deut. Akad. Wiss. Berlin, KI. Math.-Phys. Tech. 189 (1916).
[19] H. Reissner, Ann. Phys. (Leipzig) **50**, 106 (1916); G. Nordström, Proc. K. Ned. Akad. Wet. **20**, 1238 (1918).
[20] C. S. Peca and J. P. S. Lemos, Phys. Rev. D **59**, 124007 (1999); P. Mitra, Phys. Lett. B **459**, 119 (1999); S. W. Hawking and H. S. Reall, Phys. Rev. D **61**, 024014 (2000); B. Wang, E. Abdalla, and R.-K. Su, *ibid.* **62**, 047501 (2000).
[21] K. D. Kruskal, Phys. Rev. **119**, 1743 (1960).
[22] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-time* (Cambridge University Press, Cambridge, England, 1973).
[23] J. Nash, Ann. Math. **60**, 383 (1954); **63**, 20 (1956); C. J. S. Clake, Proc. R. Soc. London **A314**, 417 (1970).
[24] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).
[25] R. Laflamme, Phys. Lett. B **196**, 449 (1987).
[26] J. Demers, R. Lafrance, and R. C. Meyers, Phys. Rev. D **52**, 2245 (1995); A. Ghosh and P. Mitra, Phys. Lett. B **357**, 295 (1995); S. P. Kim, S. K. Kim, K. S. Soh, and J. H. Yee, Int. J. Mod. Phys. A **12**, 5223 (1997); G. Cognola and P. Lecca, Phys. Rev. D **57**, 1108 (1998).
[27] J. D. Brown, J. Creighton, and R. B. Mann, Phys. Rev. D **50**, 6394 (1994).
[28] R. P. Kerr, Phys. Rev. Lett. **11**, 237 (1963).
[29] Jeongwon Ho, W. T. Kim, Y.-J. Park, and H. Shin, Class. Quantum Grav. **14**, 2617 (1997); G. Clement, Phys. Rev. D **57**, 4885 (1998); G. Cognola, *ibid.* **57**, 6292 (1998); V. P. Frolov and D. V. Fursaev, Class. Quantum Grav. **15**, 2041 (1998); J.-L. Jing and M.-L. Yan, Phys. Rev. D **60**, 084015 (1999); S. Q. Wu and X. Cai, gr-qc/9907054; S. Mukohyama, Phys. Rev. D **61**, 124021 (2000); Z. Li, *ibid.* **62**, 024001 (2000).